ME-41 Project 2 Report

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1: Executive Summary

Fatigue

The safety factor for fatigue is 1.374, showing that the shaft stays under the endurance limit.

Shafts / Combined Loading

The safety factor for combined loading is 1.65. Safety factor for Goodman criteria is 2.387. Safety Factor for Langer Criteria is 3.829. This shows that the shaft will not fail, and that Goodman criteria is of most concern.

Springs

The static factor of safety for the compression spring is 1.98, showing that the spring will not fail. The dynamic safety factor was not computed due to the spring being in a static setting.

Power Screws

The safety factor for raising the screw is 1.1404, and the safety factor for lowering the screw is .6908, showing that the screw can accidentally be lowered while in operation.

Gears

The safety factor for bending is 1.4344, while the safety factor for wear is 1.1914, showing that the gear will not fail under either condition, but that wear is of more concern.

Tabular Results

Result
1.374
1.65
2.387
3.829
1.98
1.1404
.6908
1.4344
1.1914

2: Introduction

In this project, both dynamic failure methods and machine elements are analyzed on a CR Clarke injection molder. In particular, the dynamic failure methods studied were fatigue and combined loading, and the machine elements analyzed were springs, power screws, and gears. All analysis done within this project is to determine the operational safety of the injection molder under steady and persistent use. This is explicitly conducted through the determination of factors of safety that determine whether the CR Clarke injection molder will fail under certain criteria.

As opposed to Project 1, this project focuses on a more uncertain object in which key assumptions need to be made to progress with analysis. This resembles more closely the conditions seen in engineering practice, where definitions and failure criteria are up to the engineer to decide and are not given explicitly. Therefore, failure criteria in this project are often times not defined by a single property, like yielding is to the yield strength, but rather defined by a series of questions that the engineer must ask themselves before determining the failure criteria.

3: Analysis

Disclaimer: I did this report before it was made apparent to me that the following calculations are not necessary. I left all calculations in to not waste all the work I did on this project over Thanksgiving break.

3.1 Fatigue

 η

Although the use of the CR Clarke injection molder being studied is not in a high-cycle environment, the machine was designed with the ability to be used in that type of environment. Therefore, fatigue was analyzed.

Assuming that the injection molder crank is introduced to a load of 50 lb and the crank has a radius of 8.5 in, the torque being created during operation is 425 lb·in. This creates a torsion within the inner shaft of magnitude $\tau = \frac{16T}{\pi D^3} = \frac{16\cdot425}{\pi \cdot .625^3} = 8866$ psi. This torsion will have localized maximums near notches in the shaft; therefore, these concentrations must be accounted for. Using the graphs in the appendix, characteristics to determine the stress concentration factor k_{fs} can be found. Using the values extrapolated in the appendix, the stress concentration factor can be found. $k_{fs} = 1 + q_s(k_{ts} - 1) = 1 + .6(2 - 1) = 1.6$. Applying this factor to the torsion found earlier leads to a maximum torsion of $\tau_{max} = k_{fs}\tau = 1.6 \cdot 8866 = 14185$ psi.

We must relate this value to the endurance limit of the shaft, which can be found using Marin factor analysis. Marin factors for a machined shaft of diameter .625 in, at room temperature with 90% reliability are as follows. $k_a = a(S_{ut})^b = 2.70(100)^{-.265} = .7968$ $k_b = .879 \cdot d^{-.107} = .879(.625)^{-.107} = .9240$ $k_c =$.59 (torsion) $k_d = 1$ (room-temp) $k_e = .897$ (90% reliability.). Finally, assuming the Mischke correlation holds, S_e can be defined as half the ultimate strength, which is 50 kpsi. Using this, we can define $S'_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e = .7958(.9240)(.59)(1)(.897)(50 \text{ kpsi}) = 19.490 \text{ kpsi}.$

Finally, a safety factor for the endurance limit can be defined as

$$\eta = \frac{S'_e}{\tau_{max}} = \frac{19.490}{14.185} = 1.374$$

3.2 Shafts / Combined Loading

Every time that the CR Clarke injection molder is used, it is exposed to constant torsion and alternating bending as a result of the crank being used. Because of this, the shaft is exposed to combined loading. The magnitude of the torsion is the same as what was found during fatigue. The alternating bending moment has a magnitude of 51.5 lb·in and a stress of $\sigma_b = \frac{32M}{\pi d^3} = \frac{32(51.5)}{\pi (.625)^3} = 2149$ psi.

It is important to note that due to the prevalence of notches in the shafts, the values of torsion and bending will be amplified. The torsional stress concentration has been previously solved for, and the bending stress concentration factor can be defined as $k_f = 1 + q(k_t - 1) = 1 + .55(3.75 - 1) = 2.5125$, where k_t and q are found using tables found in the appendix.

Using this, a safety factor for combined loading can be expressed as follows:

$$\eta = d^3 \left[\frac{16}{\pi} \left(\frac{1}{S_e} \{4(k_f M_a)^2 + 3(k_{fs} T_a)^2\}^{1/2} + \frac{1}{S_y} \{4(k_f M_m)^2 + 3(k_{fs} T_m)^2\}^{1/2} \right) \right]^{-1}$$

= .625³ $\left[\frac{16}{\pi} \left(\frac{1}{19490} \{4(2.5125 \cdot 51.5)^2 + 3(0)^2\}^{1/2} + \frac{1}{75000} \{4(0)^2 + 3(1.6 \cdot 425)^2\}^{1/2} \right) \right]^{-1} = 1.65$

It is also important to contextualize which loading condition is more severe. To do so, we first have to define $\sigma_a = (2.5125 \cdot 2149) = 5399.4$ psi and $\sigma_m = (1.6 \cdot 8866) = 14185$ psi. Using these new parameters, we can

define factors of safety for both Goodman (fatigue) and Langer (yielding) criteria.

$$\eta_{\rm G} = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{5.399}{19.49} + \frac{14.185}{100}\right)^{-1} = 2.387, \quad \eta_{\rm L} = \left(\frac{\sigma_a}{S_y} + \frac{\sigma_m}{S_y}\right)^{-1} = \left(\frac{5.399}{75} + \frac{14.185}{75}\right)^{-1} = 3.829$$

3.3Springs

The CR Clarke injection molder relies on two compression springs to return the injection head from the mold. An analysis of compression screw failure criteria was conducted, with results shown below.

Spring Material (Pass): Music Wire, $S_{ut} = \frac{A}{d^m} = \frac{201}{.10^{.145}} = 280.67$ kpsi Music wire is the most widely used spring material, and it is available in diameters ranging from .005 in to .125 in. The diameter for this spring falls in that range.

Stability (Pass): $L_o < 2.63 \frac{D}{\alpha}$, $L_o = 3$ in, D = 1 in, $\alpha = .5$, $2.63 \frac{D}{\alpha} = 2.63 \frac{1}{.5} = 5.26$, 3 < 5.26 It should be noted that this failure will be mitigated by a shaft being placed in the middle of the spring.

Spring Index Limit (Pass): 4 < C < 12, $C = \frac{D}{d} = \frac{1}{.1} = 10$, 4 < 10 < 12

Active turn limit (Fail): $3 \le N_A \le 15$, $N_A = 17$ (from problem statement), $3 \ne 17 \ne 15$

Min Operating Length (Pass):
$$L_{min} > L_S$$
, $L_S = d(N_T + 1) = .1(19 + 1) = 2$ in, $L_{min} = 3$ in, $3 > 2$

Critical Frequency (Pass): $f_n > F_{\text{applied}}$, $f_n = \frac{1}{2}\sqrt{\frac{kg}{W}}$, $k = \frac{d^4G}{8D^3N_A} = \frac{.1^4(10\cdot10^6)}{8(1)^3(17)} = 7.35\frac{N}{m}$, $W = \frac{\pi^2 d^2 D N_a \gamma}{4} = \frac{(\pi)^2(.1)^2(1)(17)(.283)}{4} = .1187 \text{ lb}$, $f_n = \frac{1}{2}\sqrt{\frac{kg}{W}} = \frac{1}{2}\sqrt{\frac{7.35\cdot386}{.1187}} = 77.3 \text{ Hz}$

As long as the applied frequency is under $\frac{77.3}{20} = 3.865$ Hz, the spring will not fail under frequency criteria.

Static Safety Factor (Pass): $\eta > 1.2$, $\eta = \frac{S_{sy}}{\tau_{max}}$, $S_{sy} = .45S_{ut} = .45(280.67) = 126.30$ kpsi, $\tau_{max} = k_B \left(\frac{8FD}{\pi d^3}\right)$, $k_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$, $F_{max} = kx_{max} = 7.35 \cdot 3 = 22.05$ lb, $\tau_{max} = k_B \left(\frac{8FD}{\pi d^3}\right) = 1.135 \cdot \frac{8 \cdot 22.05 \cdot 1}{\pi \cdot 1^3} = 63750$ psi, $\eta = \frac{S_{sy}}{\tau_{max}} = \frac{126.30}{63.750} = 1.98$, 1.98 > 1.2Static failure criteria was used because the CR Clarke injection molder is not in a dynamic setting.

$\mathbf{3.4}$ **Power Screws**

A power screw is used to clamp the two ends of the mold together. Because of this, it is vital to determine whether the power screw is self-locking or not so that the mold does not open while injection is happening. Furthermore, it is important for the mold to not be opened accidentally while in operation. Because of this, a safety factor for accidentally opening the mold was analyzed.

Self-Locking Criteria: $\pi \cdot f \cdot d_m > \ell$, $\pi \cdot f \cdot d_m = \pi \cdot .1 \cdot .45 = .1413$, $\ell = .077$, .1413 > .077 Therefore, the power screw is self-locking.

Power-Screw Factor of Safety:

Assuming the force that the power screw resists is 8150 lb, the torque to raise and lower the power screw are: Torque to Raise = $\frac{Fd_m}{2} \left(\frac{\ell + \pi f d_m sec(\alpha)}{\pi d_m - f \ell sec(\alpha)} \right) + \frac{Ff_c d_c}{2} = \frac{8150 \cdot 45}{2} \left(\frac{.077 + \pi (.10)(.45)sec(28)}{\pi (.45) - .10(.077)sec(28)} \right) + \frac{8150(.10)(.5)}{2} = 513.22 \text{ lb} \cdot \text{in}$ Torque to Lower = $\frac{fd_m}{2} \left(\frac{\pi f d_m sec(\alpha) - \ell}{\pi d_m + f \ell sec(\alpha)} \right) + \frac{Ff_c d_c}{2} = \frac{8150 \cdot .45}{2} \left(\frac{\pi (.10)(.45)sec(28) - .077}{\pi (.45) + .10(.077)sec(28)} \right) + \frac{8150(.10)(.5)}{2} = 310.88 \text{ lb} \cdot \text{in}$ Assuming the crank for the power screw has a radius of 4.5 in, the forces to move the power screw are: Force to Raise: $\frac{T}{r} = \frac{513.22}{4.5} = 114.04$ lb Force to Lower: $\frac{T}{r} = \frac{310.88}{4.5} = 69.08$ lb

Assuming a human can supply 100 lb of force to the crank, factors of safety can be defined:

$$\eta_{raise} = \frac{F_{\text{to move}}}{F_{\text{human}}} = \frac{114.04}{100} = 1.1404, \quad \eta_{lower} = \frac{F_{\text{to move}}}{F_{\text{human}}} = \frac{69.08}{100} = .6908$$

3.5 Gears

A rack and pinyon gearset is used to lower the injection head into the mold. Based on geometry, the pinyon is often the gear that will fail in these gearsets, so the pinyon was analyzed for the two gear tooth failure methods: bending and wear. The results are seen below:

Maximum Allowable Stress from Bending: $\sigma_b = \frac{S_T Y_N}{S_F K_T K_R} = \frac{42(1.2218)}{1(1)(.85)} = 60.37$ kpsi $S_T = 42$ See appendix graph $Y_N = 1.2218$ See appendix graph $S_F = 1$ Given in problem statement $K_T = 1$ Given in problem statement

 $K_R = .85$ See appendix table

Maximum Allowable Force from Bending: $W_b = \frac{\sigma_b \cdot F \cdot J}{K_o K_v K_S P_d K_m K_b} = \frac{60.37(.8)(.27)}{1(1.044)(1)(12.8)(1)(1)} = 975.408$ lb $K_o = 1$ See Appendix Table $K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = \left(\frac{85.28 + \sqrt{60}}{85.28}\right)^{.5} = 1.044, \quad B = .25(12 - Q_v) = .5, \quad A = 50 + 56(1 - B)^{2/3} = 85.28$ $K_S = 1$ Given in problem statement $K_m = 1$ Given in problem statement $K_b = 1$ Given in problem statement J = .27 See appendix graph F = .8 Given in problem statement $P_d = 12.8$ Given in problem statement

Maximum Allowable Stress from Wear: $\sigma_c = \frac{S_C}{S_H} \frac{Z_N C_H}{K_T K_R} = \frac{121.55}{1} \frac{1.166(1)}{1(.85)} = 166.7952$ kpsi

 $S_C = 1$ See Appendix graph

 $Z_N = 1$ See appendix graph

 $C_H = 1$ Given in problem statement

 $S_H = 1$ Given in problem statement

Maximum Allowable Force from Wear:
$$W_c = (-\frac{\sigma_c}{C_p})^2 \frac{d_p FI}{K_o K_v K_S K_m C_F} = (-\frac{166795.2}{2300})^2 \frac{1.25(.8)(.161)}{1(1.044)(1)(1)(1)} = 810.2$$

 $d_p = 1.25$ Given in problem statement I = .161 Given in problem statement $C_F = 1$ Given in problem statement

Safety Factors:

The user pushes the capstain with 50 lb of force. Given that the capstain has a radius of 8.5 in, the torque being applied is 50 lb \cdot 8.5 in = 425 lb in. The torque is transmitted over the pitch radius of the pinyon gear, leading to a force of $\frac{425}{1.25/2} = 680$ lb being exerted on the pinyon. Safety factors can be defined as follows:

$$\eta_{\text{bending}} = \frac{W_b}{F_{\text{on pinyon}}} = \frac{975.408}{680} = 1.4344, \quad \eta_{\text{wear}} = \frac{W_c}{F_{\text{on pinyon}}} = \frac{810.2}{680} = 1.1914$$

It should be noted that in the homework P2 submissions, I forgot to find the pitch radius and instead found the force transmitted to the pinion using purely the pitch diameter. The calculations above have corrected this mistake.

4: Discussion

4.1 Discussion of Fatigue Criteria

Firstly, it is important to note that while the CR Clarke injection molder currently in Bray is not in a highcycle environment, many injection molders are. For instance, Lego uses injection molding to produce their bricks, so having injection molders that can withstand fatigue is paramount. That being said, if the machine is to be used in a heavy work environment, as described earlier, it is reasonable to require that the maximum load placed on the shaft be under the shaft's endurance limit, such that it can withstand infinitely many cycles. Therefore, the safety factor was defined with that in mind, and as can be seen from the analysis, the maximum load that the shaft experiences is below the endurance limit.

If the buffer between the endurance limit and the maximum load needs to be increased, the best way to achieve this is to increase the diameter of the shaft, as this will decrease the torsion that acts on the shaft. It is also important to note that this will also decrease the value of the Marin factor k_b , which, in turn, decreases the endurance limit. However, the rate at which the torsion decreases with a larger diameter far exceeds the rate at which k_b decreases with the same increased diameter. Another design choice is changing the material to one with a higher S_{ut} . Increasing the ultimate strength dramatically increases the value of S_e , which increases the endurance limit, thus creating a buffer between it and the maximum load. This, once again, comes at the cost of a decreasing Marin factor, as k_a decreases with increasing S_{ut} . Similar to increasing the diameter, the benefit of a dramatically increasing S_e mitigates the impact of a slightly decreasing k_a .

4.2 Discussion of Shafts / Combined Loading Criteria

Because there is a combination of loading types in the shaft when the capstan is pulled, one type of loading affects the performance of the shaft more dramatically. In this case, it can be seen that the Goodman (or fatigue) criteria presents an increased risk of failure compared to the Langer (or yielding) criteria. This is evident in a comparison of their safety factors, as the safety factor for the Goodman criteria is substantially lower than that of the Langer criteria. Therefore, design changes for fatigue are of the most concern.

When evaluating the factor of safety for the Goodman criteria, it can be noted that smaller ratios between the load experienced and the maximum allowable load correspond to higher safety factors. Therefore, based on the ratios shown in the analysis of the factor of safety, it can be deduced that the ratio of $\frac{\sigma_a}{S'_{e}}$ is the limiting factor. Ways of increasing the endurance limit, as described in the previous section, can be explored, or methods of decreasing σ_a can be investigated. Assuming the moment placed on the shaft must remain the same, increasing the diameter of the shaft will once again increase the safety factor, as the bending stress is inversely related to the diameter cubed.

It should also be noted that both design modifications proposed for improving the Goodman criteria also improve the combined loading condition. This can be clearly seen from the combined loading factor of safety being proportional to the diameter cubed and linearly proportional to the endurance limit $(\frac{1}{S'_{e}})^{-1} = S'_{e}$.

4.3 Discussion of Spring Criteria

There are many definitions of failure when considering a compression spring. The number of definitions, however, can be reduced if the spring is only subjected to a static load. This was the main assumption used when analyzing the compression spring in the injection molder. This somewhat contradicts the previous statement that injection molders experience high-cycle environments. However, for the purposes of the springs specifically used on a hand-cranked injection molder, as seen in the CR Clarke injection molder, it is fair to assume that the machine does not operate in a dynamic setting. This eliminates the need for a dynamic factor of safety. It also eliminates the threat of the critical frequency criteria, as a human is incapable of operating the machine at 3.865 times per second while successfully molding a part.

With that in mind, the compression spring fails by definition in one criterion: the active turns criterion $(3 \le N_A \le 15)$. While technically this criterion amounts to failure by definition, the designers of the CR Clarke injection molder have practically eliminated the failure by placing a rod in the middle of the spring. The main concern with the active turn failure criterion is that the spring will buckle due to its slenderness. This is why the rod in the middle of the spring is so important, as it 'catches' the spring if it were to buckle, effectively making the spring perfectly functional. Therefore, this criterion is not of concern.

4.4 Discussion of Power Screw Criteria

The power screw is crucial to the overall functionality of the injection molder, as it is responsible for keeping the mold in place while forces from the injection process are exerted on it. This key functionality is encapsulated in the 'self-locking' property that the power screw exhibits, ensuring that the screw stays in place unless acted upon by an external force. Unfortunately, there is an easy way to act on the power screw, leading to the CR Clarke injection molder's most significant failure: a human accidentally being able to move the power screw, and in turn, the mold. It should be noted that, assuming a human can only exert 100 lb, the power screw can only be lowered, not raised, while in operation. However, the magnitude of its failure is quite dramatic.

There are two methods to improve the operational safety of the power screw. The simplest is decreasing the radius of the power screw handle. Decreasing this radius reduces the amount of torque that a human can provide. However, the radius would have to decrease quite dramatically. To ensure that the power screw does not fail, the radius of the handle must be reduced from 4.5 in to 3.1 in. This may defeat the ergonomic effectiveness of the handle, making it undesirable. The other method of ensuring operational safety is by increasing the torque needed to lower the screw. From the equation for torque, it can be seen that, in the case of lowering the screw specifically, the collar friction provides more resistance than the friction from the threads. Therefore, increasing the diameter of the collar can further compound this resistance. By increasing the collar diameter to 1 in, the threat of failure can be eliminated entirely (calculations can be found in the appendix).

4.5 Discussion of Gear Criteria

In rack and pinion setups, it is often seen that the pinion fails far before the rack due to its geometry; therefore, the pinion was evaluated for failure instead of the rack. From the analysis of the two main failure modes of gear teeth—bending stress and wear—it can be seen that the pinion is not a concern for failure, as both failure modes boast safety factors greater than one. It can still be noted that wear is more of a concern than bending.

To create more buffer room from failure due to wear, increasing the pitch diameter can lead to favorable results, as increasing it slightly (from 1.25 in to 1.5 in) can make the factor of safety for wear match that of bending. Increasing the size of the gear tooth face will also positively impact the safety factor for both wear and bending, leading to an improvement in both gear tooth safety criteria.

Once again, these safety factors do not indicate failure, so changes to the gear setup are not as pressing as other design changes.

5: Appendix

5.1 Graphs for 3.1 Fatigue



Extrapolating yields values of $k_{ts} = 2.0$, and $q_s = .6$

5.2 Graphs for 3.2 Shafts / Combined Loading



Extrapolating yields values of $k_t = 3.75$, and q = .55



Plot showing Goodman and Langer criteria with Load Line

5.3 Calculation for Power Screw Factor of Safety

Assuming that the collar of the power screw is increased to 1 inch, yields torques of: Torque to Raise = $\frac{Fd_m}{2} \left(\frac{\ell + \pi f d_m sec(\alpha)}{\pi d_m - f \ell sec(\alpha)} \right) + \frac{Ff_c d_c}{2} = \frac{8150 \cdot 45}{2} \left(\frac{.077 + \pi (.10)(.45)sec(28)}{\pi (.45) - .10(.077)sec(28)} \right) + \frac{8150(.10)(1)}{2} = 716.97 \text{ lb} \cdot \text{in}$ Torque to Lower = $\frac{fd_m}{2} \left(\frac{\pi f d_m sec(\alpha) - \ell}{\pi d_m + f \ell sec(\alpha)} \right) + \frac{Ff_c d_c}{2} = \frac{8150 \cdot .45}{2} \left(\frac{\pi (.10)(.45)sec(28) - .077}{\pi (.45) + .10(.077)sec(28)} \right) + \frac{8150(.10)(1)}{2} = 514.65 \text{ lb} \cdot \text{in}$ Which increases the forces required to move. These forces are given below assuming that the crank radius stays the same. Force to Raise: $\frac{T}{r} = \frac{716.97}{4.5} = 159.32 \text{ lb}$ Force to Lower: $\frac{T}{r} = \frac{514.65}{4.5} = 114.36 \text{ lb}$

Assuming a human can supply 100 lb of force to the crank, new factors of safety can be defined:

$$\eta_{raise} = \frac{F_{\text{to move}}}{F_{\text{human}}} = \frac{159.32}{100} = 1.5932, \quad \eta_{lower} = \frac{F_{\text{to move}}}{F_{\text{human}}} = \frac{114.36}{100} = 1.1436$$

5.4 Graphs for 3.5 Gears

5.4.1 Graphs and Tables for Allowable Bending Stress



Left - $S_T = 42$ — Middle - Stress Cycle Factor $Y_N = 1.2218$ — Right - Reliability Factor $K_R = .85$



Left - Overload factor $K_o = 1$ — Right - Geometry factor J = .27

5.4.3 Graphs and Tables for Allowable Wear Stress



Left - $S_C = 121.55$ — Right - Stress Cycle Factor $Y_N = 1.166$