

ME-41 Project 1 Report

William Cusato



November 13, 2024

1: Exectutive Summary

1.0.1 Deflection

Theoretical (.7722 or 0.80966), experimental (.8106) and SolidWorks (0.9459) safety factors are below 1.0, indicating that the c-shaped spring will fail under deflection.

1.0.2 Yielding:

Tresca and Von-Mises Criteria: Safety factors for the three locations with Tresca criteria are 1 - 1.5615, 2 - 1.535, 3 - 1.1702. Safety factors for the three locations with Von-Mises criteria are 1 - 1.5617, 2 - 1.535, 3 - 1.300. Both methods show acceptable safety margins.

Experimental Criteria: The safety factor gathered from the Instron experiment is 1.07448, suggesting that the c-shaped spring approaches the yield-limit.

SolidWorks Criteria: The localized yielding safety factors from SolidWorks vary more drastically between locations (1.014 to 1.543), suggesting localized areas may approach yield limits. However, when the whole c-shaped spring was taken into consideration, yield limits were exceeded, leading to a safety factor of .93492.

1.0.3 Tabular Results

Factor of Safety	Result
Deflection, E = 10000 ksi	.80966
Deflection, E = 9540 ksi	.7722
Tresca - 1	1.5615
Tresca - 2	1.535
Tresca - 3	1.1702
Von-Mises - 1	1.5617
Von-Mises - 2	1.535
Von-Mises - 3	1.300
Experimental Deflection	.8106
Experimental Yielding	1.0747
SolidWorks Deflection	.9459
Overall SolidWorks Yielding	.93492
SolidWorks Yielding - 1	1.041
SolidWorks Yielding - 2	1.543
SolidWorks Yielding - 3	1.014

2: Introduction

In this project, the static failure methods of a c-shaped spring that is to be used on a Mars Lander were analyzed. The two failure methods, deflection and yielding, will be analyzed with respect to a 75-lb load located on the far end of the c-shaped spring. Furthermore, the spring is simply supported on the bottom leg. Failure analysis is provided by three main methods: analytical, simulation, and experimental. Analytical results of deflection are calculated through the use of Castigliano's method, and analytical results of yielding are calculated through the use of Mohr's circle, Tresca and Von-Mises failure criteria. Simulation is provided through Solidworks. Finally, experimental data was compiled using an Instron Tensile tester that provides a quasi-static compressive load on the same location as the analytical and simulation loads were placed. A cross-analysis of the failure criteria was conducted to find the validity of the results.

3: Analysis

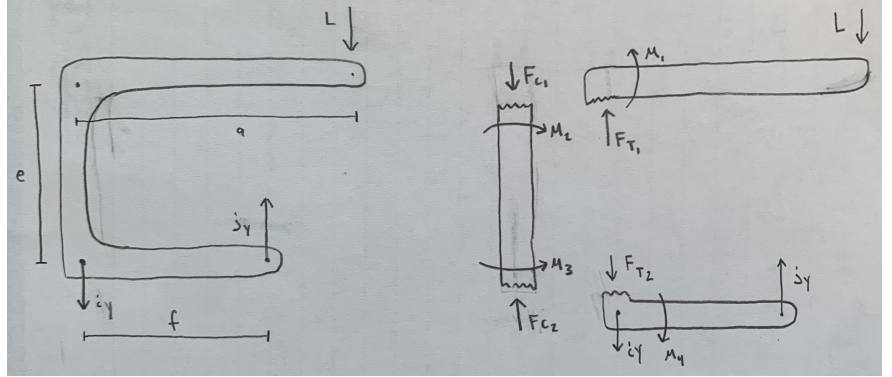
3.1 Free-Body Diagram

$$L = F_{T1} = F_{T2} = F_{C1} = F_{C2} = 75 \text{ lbf}$$

$$M_1 = M_2 = M_3 = M_4 = La = 525 \text{ lbf} \cdot \text{in}$$

$$j_y = \frac{La}{f} - L = \frac{75 \cdot 7}{4} - 75 = 56.25 \text{ lbf}$$

$$j_y = \frac{La}{f} = \frac{75 \cdot 7}{4} = 131.25 \text{ lbf}$$



3.2 Moment Equations and Castigliano's Calculation

Top Horizontal Leg: The moment, M_1 , must counteract the imposed moment from load L completely, which yields a value for moment of $L \cdot a$. Define x to be the distance from the reaction moment to load L . The moment equation along the top horizontal leg is $L \cdot a - L \cdot x = L(a - x)$. The leg length is the entire span of the top horizontal leg. These parameters can be used to find deflection using Castigliano's method.

$$\delta = \int \frac{1}{EI} (M \frac{\partial M}{\partial x} dx) = \int_0^a \frac{1}{EI} L(a - x)(a - x) dx = \frac{La^3}{3EI}$$

Finally, substituting $I = \frac{tb^3}{12}$ yields $\delta = \frac{4La^3}{Etb^3}$

Middle Vertical Leg: It is evident from the free-body diagram that there will exist two components of deflection in this leg because of the two different load methods. Starting with compression, Hooke's law can be used to find the deflection. The compressive load must be equal to the load L , such that each individual leg is kept in equilibrium. Also, the leg length is equal to the length of the entire vertical section. Using these parameters, deflection can be calculated.

$$U = \int_0^e \frac{F^2}{2AE} dx = \frac{F^2 x}{2AE} \Big|_0^e = \frac{L^2 e}{2AE} \quad \frac{\partial U}{\partial L} = \delta = \frac{Le}{AE}$$

Finally, substituting $A = tc$ yields $\frac{Le}{tcE}$.

The vertical leg experiences a constant moment equal to La . This is due to the lack of non-axial loads placed on it, and the fact that M_2 and M_3 are equal and opposite. Because analysis is on the vertical leg, the length scale of the vertical leg is to be used. With these parameters, deflection can be calculated.

$$\delta = \int \frac{1}{EI} (M \frac{\partial M}{\partial x} dx) = \int_0^e \frac{1}{EI} La^2 dx = \int_0^e \frac{L}{EI} a^2 dx = \frac{L}{EI} (a^2 x) \Big|_0^e = \frac{La^2 e}{EI}$$

Finally, substituting $I = \frac{tc^3}{12}$ yields $\delta = \frac{12La^2 e}{Etc^3}$

Bottom Horizontal Leg: Define x to be in the same direction as in the top horizontal leg. The moment M_4 , must be able to counteract the moment created by j_y completely, with that moment being equal to $j_y \cdot f$. This yields a moment equation of $La - \frac{La}{f} x = La(1 - \frac{x}{f})$, and the length scale is that of the bottom horizontal leg. Once again, these parameters are used in Castigliano's theorem to find deflection.

$$\delta = \int \frac{1}{EI} (M \frac{\partial M}{\partial x} dx) = \int_0^a \frac{1}{EI} La(1 - \frac{x}{f}) a(1 - \frac{x}{f}) dx = \frac{La^2 f}{3EI}$$

Finally, substituting $I = \frac{td^3}{12}$ yields $\delta = \frac{4La^2f}{Etd^3}$

Superposition: Using superposition to sum our data yields a final result of:

$$\delta = \frac{4La^3}{Etb^3} + \frac{Le}{tcE} + \frac{12La^2e}{Etc^3} + \frac{4La^2f}{Etd^3}$$

Using values of $L = 75$ lbs, $a = 7$ in, $e = 3$ in, $f = 4$ in, $t = .25$ in, $b = .75$ in, $c = .75$ in, $d = .65$ in, $E = 10000000$ psi, yields a final result of .3088 in. If we are to use $E = 9540000$ psi instead, the deflection is .32369 in.

For determination and discussion of deflection safety factors, see Section 5: Discussion.

3.3 Mohr's Circle Analysis

There are three specific locations of the c-shaped spring on which Mohr's circle analysis was conducted. These locations were (1 - the inner fillet between the vertical leg and the top horizontal leg), (2 - the middle of the inner side of the vertical leg), and (3 - the inner fillet between the vertical leg and the lower horizontal leg). These locations were chosen due to the prevalence of stress concentrations.

The Mohr's circle criteria that are most prevalent to the c-shaped spring are the principal stresses and the extreme shear, as these values help define factors of safety. They can be calculated using the following equations:

$$\sigma_{principal} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \tau_{extreme} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

For location A, $\sigma_x = -\frac{6La}{tb^2} = -\frac{6 \cdot 75 \cdot 7}{.25 \cdot .75^2} = -22400$, $\sigma_y = 0$, $\tau_{xy} = \frac{L}{bt} = \frac{75}{.75 \cdot .25} = 400$.

This yields:

$$\sigma_{1,2} = \frac{-22400}{2} \pm \sqrt{\left(\frac{-22400}{2}\right)^2 + 400^2} = 7.14 \text{ and } -22407.14 \text{ psi}, \quad \tau_{extreme,a} = \pm \sqrt{\left(\frac{22400}{2}\right)^2 + 400^2} = \pm 11207.14$$

For location B, $\sigma_x = 0$, $\sigma_y = \frac{-L}{ct} - \frac{6La}{tc^2} = \frac{-75}{.75 \cdot .25} + \frac{-6 \cdot 75 \cdot 7}{.25 \cdot .75^2} = -22800$, $\tau_{xy} = 0$.

This yields:

$$\sigma_{1,2} = \frac{-22800}{2} \pm \sqrt{\left(\frac{-22800}{2}\right)^2} = 0 \text{ and } -22800 \text{ psi}, \quad \tau_{extreme,b} = \pm \sqrt{\left(\frac{22400}{2}\right)^2} = \pm 11400 \text{ psi}$$

For location C, $\sigma_x = -\frac{6La}{tc^2} = -\frac{6 \cdot 75 \cdot 7}{.25 \cdot .75^2} = -22400$, $\sigma_y = -\frac{6La}{td^2} = -\frac{6 \cdot 75 \cdot 7}{.25 \cdot .65^2} = -29822.5$, $\tau_{xy} = -\frac{La}{fdt} = -\frac{75 \cdot 7}{4 \cdot .65 \cdot .25} = -807.69$ psi.

This yields:

$$\sigma_{1,2} = \frac{-22400 - 29822.5}{2} \pm \sqrt{\left(\frac{-22400 + 29822.5}{2}\right)^2 + 807.69^2} = -22313.12 \text{ and } -29909.37 \text{ psi},$$

$$\tau_{extreme,c} = \pm \sqrt{\left(\frac{-22400 + 29822.5}{2}\right)^2 + 807.69^2} = \pm 3798.12 \text{ psi}$$

For determination and discussion of yielding safety factors, see Section 5: Discussion.

3.4 Finite-Element Analysis Model

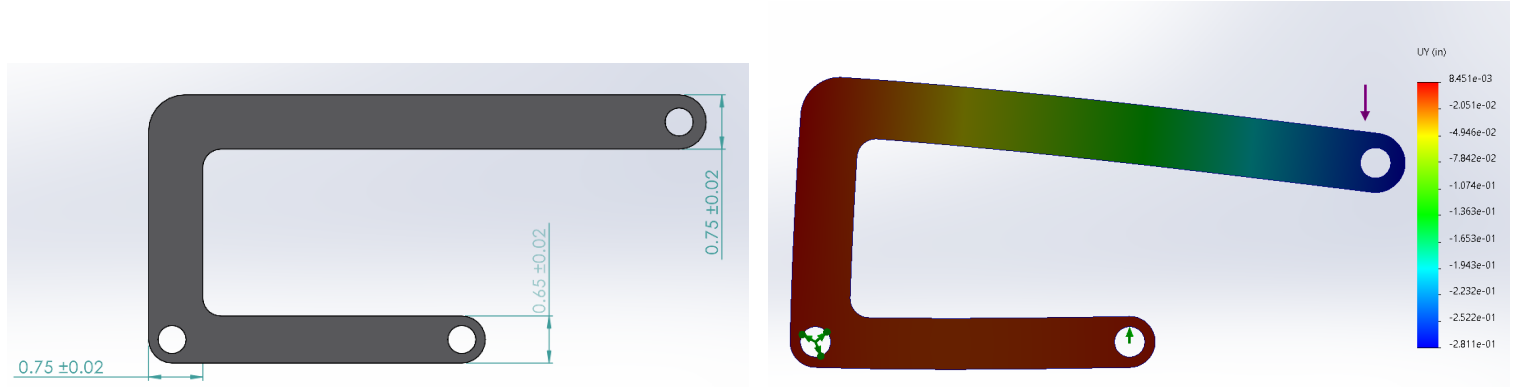
3.4.1 SolidWorks Model and Set-up:

In SolidWorks, a finite element analysis was run to compare simulative results with analytic results. First, a model of the c-shaped spring had to be made. Using the dimensions given for a last name starting with 'C,'

along with the shared dimensions, a model of the c-shaped spring was made in SolidWorks.

To simulate the support system as accurately as possible, advanced fixtures were used. A fixture on the left mounting hole was used to simulate a pin support such that there can be no translation in the x or y direction, but rotation around the pin is supported. On the right mounting hole, a roller was simulated such that only translation in the y-direction was restricted.

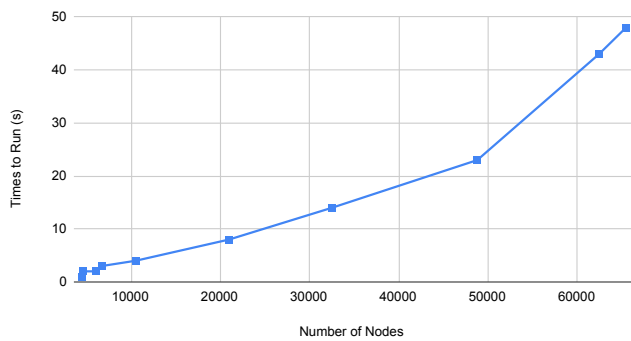
A load of 75 lbs was placed on the mounting hole located on the upper horizontal bar. The load is placed strictly in the negative y-direction.



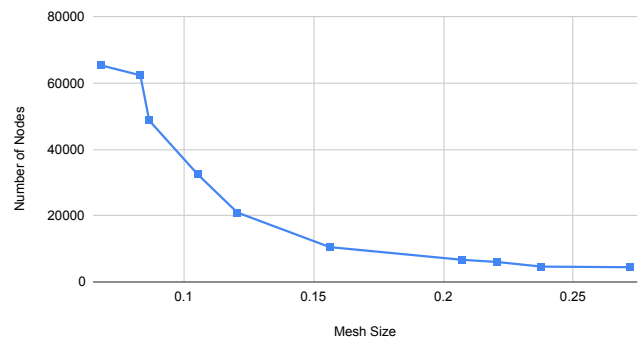
3.4.2 Mesh-Convergence:

A mesh convergence analysis was run with the goal of finding the optimally sized mesh that both maximizes performance and minimizes run time. To do this, many mesh sizes were chosen and run. Mesh size, number of mesh elements, run time, deflection, and max stress were all recorded. The results of the study can be seen below:

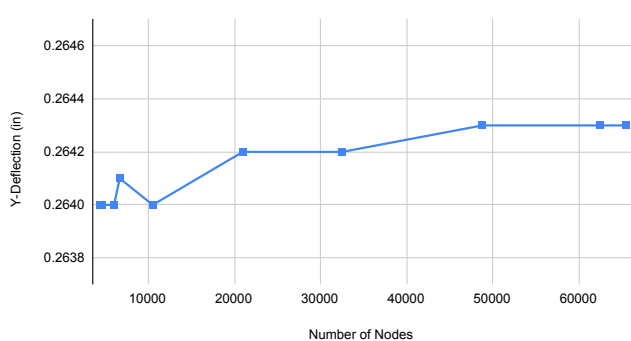
Time to Run (s) vs. Number of Nodes



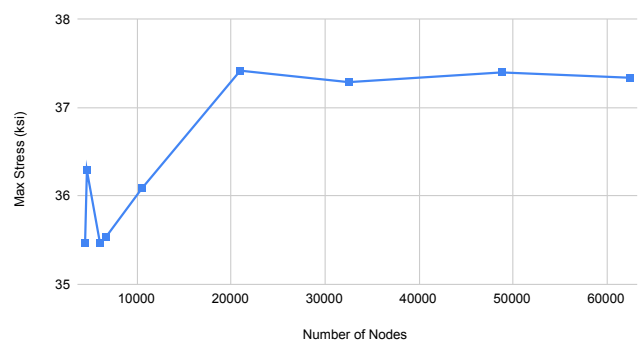
Number of Nodes vs. Mesh Size (in)



Y-Deflection (in) vs. Number of Nodes



Max Stress (ksi) vs. Number of Nodes



For detailed results of the SolidWorks simulations, see Section 4: Test and Simulation Results.

3.5 Dynamic Failure Methods

While the main subject of this project was to analyze static failure methods, select dynamic failure methods were analyzed. In particular, fracture and buckling were analyzed.

In reference to fracture, the mounting holes were analyzed to be cracks that have the chance to propagate and create failure. As such, forces were defined for the three locations previously analyzed with Mohr's circle. From the forces, stresses were evaluated, and these stresses were related to the stresses needed for uncontrollable crack growth to determine a safety factor.

In reference to buckling, it was noticed that buckling could arise in the vertical section from two means. One by the compressive force that creates buckling perpendicular to dimension t , and the other created by the imposed moment on the vertical section, which creates eccentric buckling parallel to dimension t .

In both dynamic failure criteria, the safety factors analyzed were well above one, indicating that dynamic failure is not a concern for the c-shaped spring under a 75 lbf load.

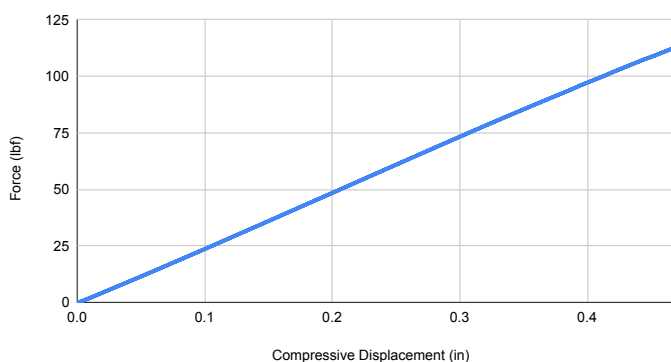
4: Test and Simulation Results

4.1 Experimental Results

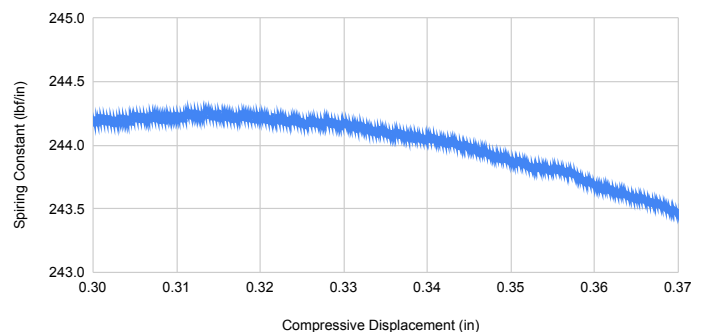
It is often observed that analytical solutions, simulations and experiment don't match each other. This can be due to a multitude of factors, such as slightly differing material properties and differences in experimental setup, among many others. Because of this, comparing results is necessary so that the most accurate perceptions of what will happen in reality are reached.

An experiment was run on an Instron tensile and compressive machine to compare experimental to analytical solutions. A load cell was used to place a quasi-static compressive load on the location of point L . Shown below are two graphs derived from the Instron data. The first depicts the force of the compressive load (lbf) to the compressive displacement (in). The second depicts the relationship between spring constant (lbf/in) and compressive displacement (in) over a concise range.

Force (lbf) vs. Compressive Displacement (in) Section 2



Spring Constant (lbf/in) vs. Compressive Displacement (in) Section 2



Left - Compressive Force (lbf) vs. Compressive Deflection (in)

Right - Spring Constant (lbf/in) vs. Compressive Displacement (in)

From the first graph, Hooke's law can be used to determine the spring constant of the c-shaped spring, k . In the case of this plot, k is the slope of the line. Using two points that are representative of the data, the slope can be determined.

$$k = \frac{\Delta y}{\Delta x} = \frac{105.7506 - 6.1372}{.4399 - .0304} = 243.26 \frac{lbf}{in}$$

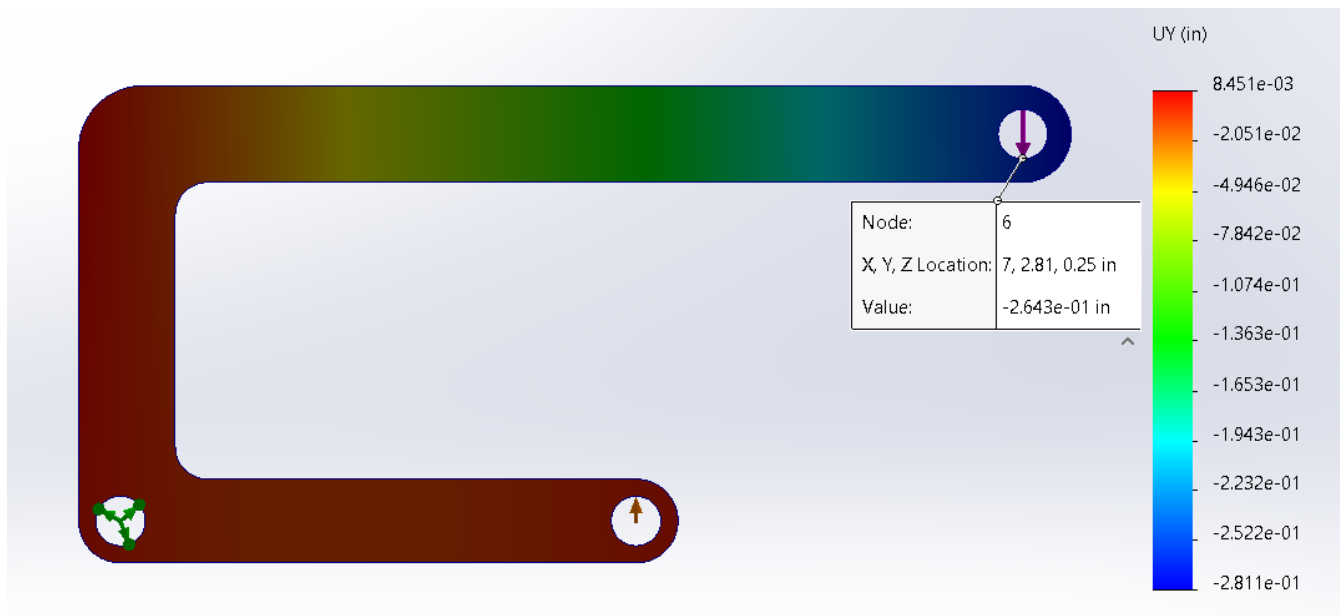
The second graph can also help us determine the spring constant. We can see that the spring constant is constant on the left-hand side of the second graph. This represents the region where yielding is not present. The value of the (roughly) horizontal line is the spring constant of the c-shaped spring, which falls around $244.25 \frac{lb_f}{in}$. The agreement between both methods of determining the spring constant is a testament to its validity.

The graph on the right-hand side can also be used to determine the yield point. As aforementioned, yielding is signified by the change of the spring constant from a horizontal line to a downward sloping curve. In the case of the graph shown above, this transition happens at a deflection value of $\delta = .33$ in. Using this deflection, we can define the force needed to yield as follows:

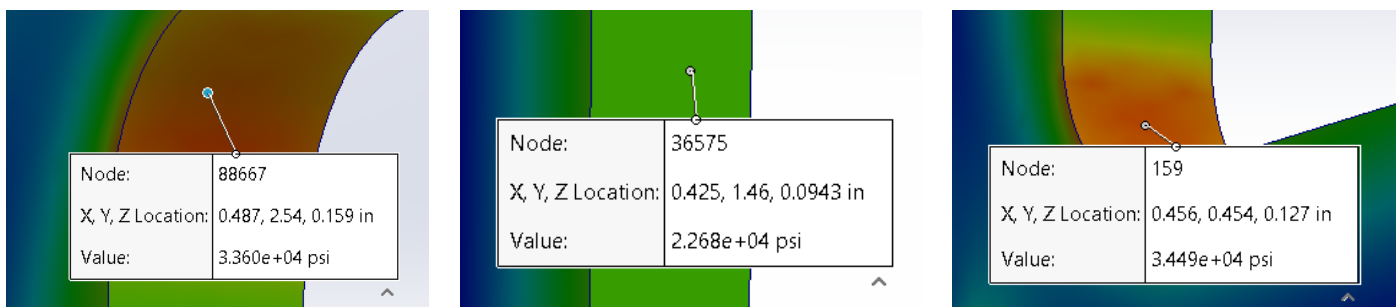
$$F_{yield} = k_{yield} \cdot \delta_{yield} = 244.25 \frac{lb_f}{in} \cdot .33 in = 80.603 lb_f$$

4.2 Simulative Results

Using the SolidWorks simulation run on the c-shaped spring, the maximum deflection and the stresses at locations 1, 2, and 3 can be determined. The maximum deflection was extrapolated using the probe feature, as were the local stresses at locations 1, 2, and 3. Results are shown below:



Deflection deformed-plot showcasing maximum deflection = .2643 in

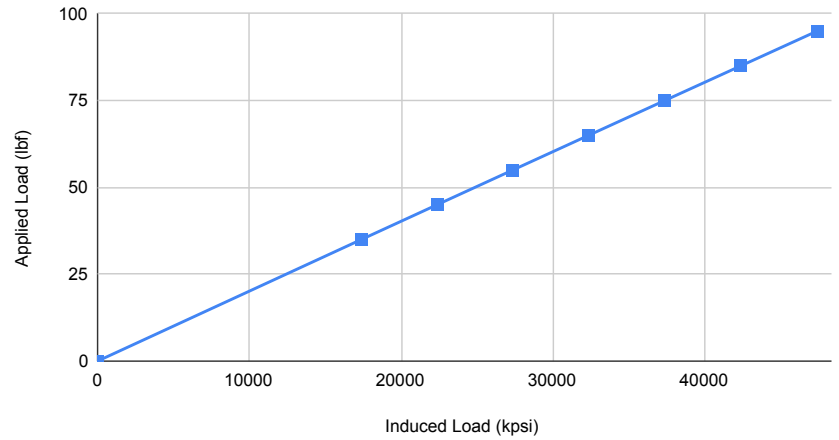


$$\sigma_1 = 33600 \text{ psi}, \sigma_2 = 22680 \text{ psi}, \sigma_3 = 34490 \text{ psi}$$

While the preceding probe values give localized values of stress, these values don't represent the stress of the c-shaped spring as a whole and thus may not give the full picture of yielding if the spring faces it. Therefore, an alternative approach to finding the yield limit was used. By varying the loads the c-shaped spring faces and plotting it against the maximum stress induced in the spring, the load that correlates to the yield strength can be extrapolated using the result.

Extrapolating the load value that corresponds to the stress value of $\sigma = 35000$ psi, it can be found that the force of yielding is $F = 70.119$ lbf.

Applied Load (lbf) vs. Induced Stress (kpsi)



5: Discussion

5.1 Factor of Safety Calculations:

5.1.1 Deflection Factor of Safety

There is no inherent limit to the amount of deflection the c-shaped spring can handle before failure, so the limit must be set by the analyzer. As such, a deflection limit of .25 inches was set to determine failure. Seeing as this limit is not a force or stress, it cannot be used to determine a factor of safety. Therefore, the corresponding load that creates a deflection of .25 inches must be found and related to the load that is being applied to the c-shaped spring. This load can be found using the superposed deflection equation.

$$\delta = \frac{4La^3}{Et b^3} + \frac{Le}{tcE} + \frac{12La^2e}{Etc^3} + \frac{4La^2f}{Etd^3} = \frac{L}{10000000 \cdot .25} \left(\frac{4 \cdot 7^3}{.75^3} + \frac{3}{.75} + \frac{12 \cdot 7^2 \cdot 3}{.75^3} + \frac{4 \cdot 7^2 \cdot 4}{.65^3} \right)$$

Solving for L yields $L = 60.725$ lbs. If we are to substitute $E = 9540000$, the corresponding value of L is 57.92 lb. These values can be used to define a factor of safety for deflection.

$$\eta_{\delta, 10000 \text{ ksi}} = \frac{L_{max}}{L} = \frac{60.725}{75} = .80967, \quad \eta_{\delta, 9540 \text{ ksi}} = \frac{L_{max}}{L} = \frac{57.92}{75} = .7722$$

Experimental Test Deflection Criteria: An experimental value of the spring constant was determined in the Instron experiment. Spring constant and force are related directly by deflection in the elastic regime. Because significant yielding is not present, it can be assumed that this relationship still exists. Therefore, a factor of safety can be written as follows:

$$F_{fail, test} = k_{test} \cdot \delta_{fail} = 243.26 \cdot .25 = 60.815, \quad \eta = \frac{F_{fail}}{L} = \frac{60.815}{75} = .8106667$$

SolidWorks Deflection Criteria: Cross multiplication can be used to find the force required to give a deflection of .25 in for the SolidWorks experiment.

$$\frac{\delta_{fail}}{\delta_{SolidWorks}} = \frac{.25}{.2643} = \frac{F}{75}, \quad F = 70.94 \text{ lbf}, \quad \eta = \frac{F}{75} = \frac{70.94}{75} = .9459$$

5.1.2 Yielding Factor of Safety

Unlike determining a factor of safety for deflection, yielding has a predetermined failure point that is characterized by the material's yield strength. In the case of the c-shaped spring, aluminum 6061-T6511 has a yield strength of 35,000 psi.

Furthermore, there are several yielding criteria to choose from, with some being more conservative than others. The two failure criteria that are pertinent to the c-shaped spring are the Tresca and Von Mises yielding criteria.

Tresca Criteria: With Tresca criteria being more conservative than Von-Mises criteria, the factors of safety for Tresca criteria should be slightly lower or equal to that of Von-Mises criteria.

The Mohr's circle for location 1 straddles the y-axis, meaning that the load line for this case exists in the second quadrant of the Tresca Criteria chart.

The Mohr's circle for location 2 is strictly non-positive, meaning that the load line for this case exists in the third quadrant of the Tresca Criteria chart.

The Mohr's circle for location 3 is strictly negative, meaning that the load line for this case exists in the third quadrant of the Tresca Criteria chart. It should be noted that a conservative value for Tresca analysis is required. Therefore, it was evaluated with respect to the largest implied Mohr's circle.

With specific criteria in mind for each location, factors of safety can be defined as the following:

$$\eta_1 = \frac{\frac{S_y}{2}}{\tau_{max}} = \frac{\frac{35000}{2}}{11207.14} = 1.5615 \quad \eta_2 = \frac{S_y}{\sigma_{max}} = \frac{35000}{22800} = 1.535 \quad \eta_3 = \frac{S_y}{\sigma_{max}} = \frac{35000}{29909.123} = 1.170$$

Von-Mises Criteria: Von-Mises failure criteria defines an ellipse that surrounds the Tresca criteria. All load conditions within the ellipse will have a factor of safety greater than one, and all load conditions outside of the ellipse will fail. The boundary of the ellipse can be represented using the following equation:

$$\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{\frac{1}{2}}$$

At location 1, $\sigma'_1 = (7.14^2 + 7.14 \cdot -22407.14 + -22407.14^2)^{\frac{1}{2}} = 22410.71$ psi

At location 2, $\sigma'_2 = (0^2 + 0 \cdot -22800 - 22800^2)^{\frac{1}{2}} = 22800$ psi

At location 3, $\sigma'_3 = (-22312.8^2 + -22312.8 \cdot -29909.123 + -29909.123^2)^{\frac{1}{2}} = 26926.96$ psi

These values can be used to define factors of safety:

$$\eta_1 = \frac{S_y}{\sigma'_1} = \frac{35000}{22410.7} = 1.5617, \quad \eta_2 = \frac{S_y}{\sigma'_2} = \frac{35000}{22800} = 1.535, \quad \eta_3 = \frac{S_y}{\sigma'_3} = \frac{35000}{26926.96} = 1.300$$

Overall Yielding from Instron Test and SolidWorks: A factor of safety for yielding can be defined using the test results extrapolated previously. The factor of safety is defined as the following:

$$\eta = \frac{F_{yield}}{F_{applied}}, \quad \eta_{Instron} = \frac{80.603}{75} = 1.0747, \quad \eta_{SolidWorks} = \frac{70.119}{75} = .93492$$

Localized Yielding Criteria using SolidWorks Probe: A factor of safety for yielding can be defined using the simulative results given by Solidworks. The factor of safety is defined as the following:

$$\eta = \frac{S_y}{\sigma_{solidworks}}, \quad \eta_1 = \frac{35000}{33600} = 1.041, \quad \eta_2 = \frac{35000}{22680} = 1.543, \quad \eta_3 = \frac{35000}{34490} = 1.0147$$

5.1.3 Discussion of Deflection Factor of Safety Results:

The factor of safety calculations for deflection show that the c-shaped spring fails in deflection, as the force exerted on the spring exceeds the force needed to achieve a deflection of .25 in. The failure can also be seen

very bluntly by observing that the deflection when the c-shaped spring is exposed to 75 lb is greater than that of the imposed deflection failure limit.

As aforementioned, it can be seen from all factor of safety calculations for deflection that the c-shape spring fails. However, the magnitude of these failures still varies from .8096 at the minimum to .9459 at the maximum. The main reason for this difference are the differences in support methods used between the three analysis methods. In particular, in the analytical results, the bottom horizontal beam is treated as a cantilevered beam with a load applied at the end. This can be interpreted through the deflection equation for the bottom horizontal beam resembling that of a cantilevered beam. This behavior is different than what is seen in the simulative and experimental results, as in these results there exists a pin at the left mounting hole that resists all deflection. Therefore, the analytical deflection equations are fundamentally different from what is seen in the other methods of examining the c-shaped spring. The reason why the Instron test result matches the analytical result for the factor of safety so closely is due to a difference in the Young's modulus. We assumed a value of $E = 10000000$ psi for all calculations, but the Young's Modulus was closer to $E = 9540000$ in the test sample. This makes deflection greater in the test sample.

5.1.4 Discussion of Yielding Factor of Safety Results:

Factor of safety results generally show that the c-shaped spring will not fail under yielding; however, the overall simulative factor of safety indicates failure. This heightened factor of safety can once again be boiled down to the constraints put on the bottom horizontal leg, as being confined to no translation induces more reaction forces than were labeled in our free-body diagram, which, in turn, induces more localized stresses. These stresses, combined with the induced stresses from the load, make the C-shaped spring yield prematurely.

In the analytical and local simulative results, it should be noted that failure at location 3 is of the most concern, as the factor of safety is substantially closer to one than the other locations under both the Tresca and Von-Mises criteria. This is due to location 3 being situated adjacent to dimension d, which is smaller than the other corresponding dimensions (b and c). This smaller area makes it so the forces in the y-direction are distributed over a smaller cross-sectional area, which makes the stresses larger. There is also a stress concentration in this area, which leads to a discrepancy between the simulative and test results against the analytical results.

Comparing the factors of safety, it can be noted that, generally, the simulative and test results are much closer to yielding than the analytical results. This in particular happens at locations 1 and 3, where there are buildups of stress concentration. It's important to note that the Mohr's circle analysis did not account for stress concentration factors when calculating the final stress result. Because of this, the stresses reported are lower in magnitude than what they should be. This accounts for the drastic difference in factors of safety between the simulative and analytical results. It should also be noted that location 2, in which there would be no build-up of stress concentrations, had factors of safety that were roughly the same between all calculations. This further alludes to the fact that stress concentration factors are needed to achieve more accurate results.

5.1.5 Design Modifications

There are many avenues that one can take to reduce the deflection of the c-shaped spring, and they can be interpreted from the superposed deflection given by:

$$\delta = \frac{4La^3}{Etb^3} + \frac{Le}{tcE} + \frac{12La^2e}{Etc^3} + \frac{4La^2f}{Etd^3}$$

Assuming that the load of 75 lbs must remain the same, there are two main design choices that can decrease the deflection experienced. The first of these being changing the material. In particular, changing the material to something with a larger Young's modulus will decrease deflection. The value for Young's Modulus needed

to not fail by deflection is given by solving for E in the above equation, which yields a value of 12350740 psi (only applicable to analytical results).

The second design choice that can decrease deflection is by changing the dimensions of the c-shaped spring. In particular, decreasing the size of dimensions a , e , and f will decrease the deflection, and increasing the value of dimensions t , b , c , and d will decrease the deflection.

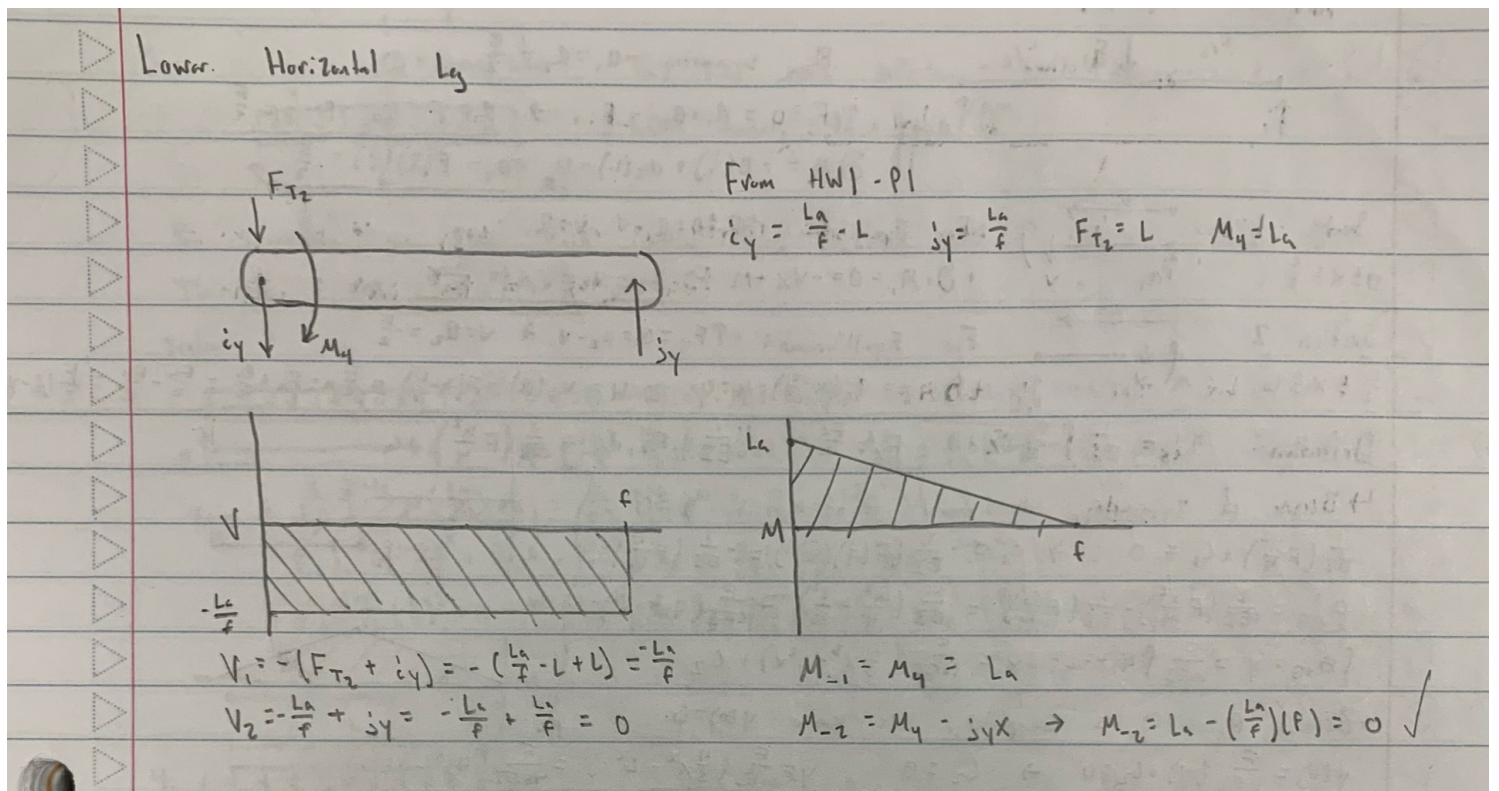
To decrease the concentration of stresses, different avenues can once again be taken. However, some are only pertinent to a couple of locations. In particular, in locations 1 and 3, there are stress concentrations due to the change in direction of stress around the bends of the c-shaped spring. To decrease the stresses at these locations, fillets were put in place, and by increasing the fillet size even more, the stress concentrations could be reduced further.

Similarly to this, stress concentrations also build due to obstacles that block the flow of stress. In the case of the c-shaped spring, these obstacles manifest themselves in the mounting holes. The larger the mounting hole, the larger the stress concentration will be, as the stress has to coalesce in a smaller area. Therefore, by decreasing the size of the mounting hole, the stress concentrations will shrink as well.

Similarly to mitigating the deflection, changing the size of certain dimensions will also decrease the stress experienced; however, only changing the dimensions that determine cross-sectional area will change the stresses experienced. Therefore, increasing the value of t will decrease stress everywhere on the c-shaped spring, and increasing the value of either b , c , or d will decrease the stresses local to the dimension.

6: Appendices

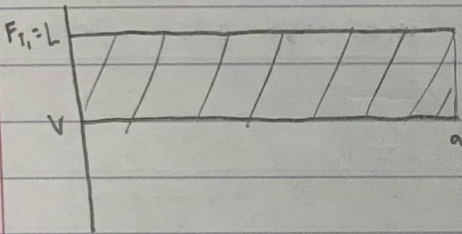
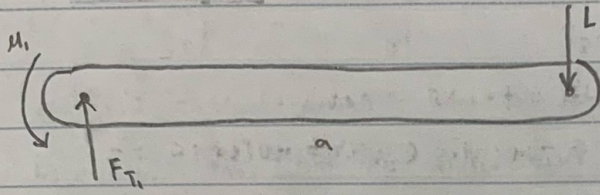
6.1 Shear / Moment Diagrams



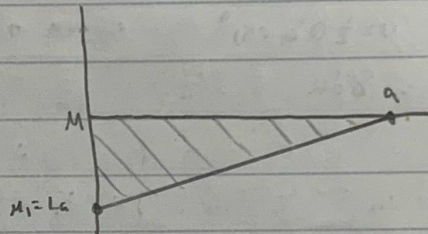
Top Horizontal Leg:

From HWI-P1:

$$F_{T1} = L \quad M_1 = La$$



$$V_1 = F_{T1} = L$$



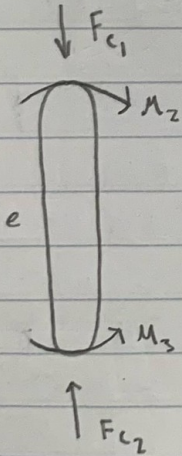
$$M_1 = La$$

$$M_2 = La - L(x) \rightarrow La - La = 0 \quad \checkmark$$

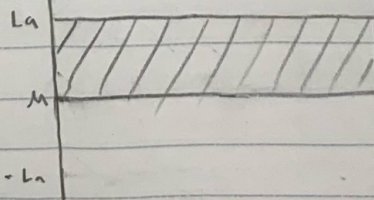
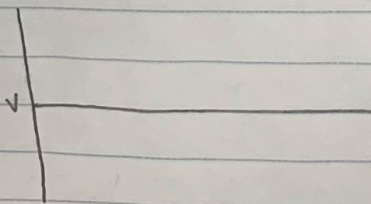
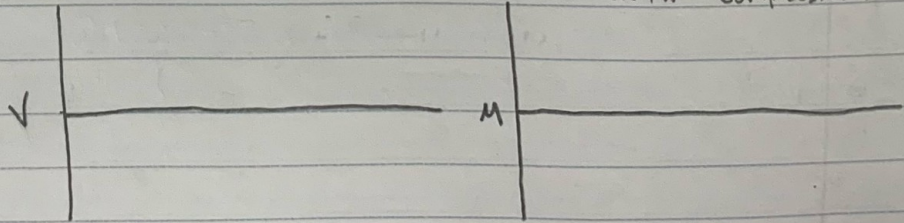
Middle Vertical Leg:

From HWI-P1

$$F_{c1} = F_{c2} = L \quad M_2 = M_3 = La$$



* For Compression *



For Bending

* For Bending Moment *

$$M_{-1} = M_2 = La$$

$$M_{-2} = M_2 - M_3 = La - La = 0 \quad \checkmark$$

6.2 Reaction Force and Moment Calculations

* Assume weight of C-shaped spring is negligible *

$$+\uparrow F_y = -L + j_y - i_y = 0 \quad i_y = \frac{L}{F} - L$$

$$+\rightarrow F_x = i_x - j_x = 0 \quad i_x = j_x = 0$$

$$+\circlearrowleft M_z = j_y(F) - L a \quad j_y = \frac{L a}{F}$$

$$+\circlearrowleft M = 0 = -L a + M_1 = 0 \quad M_1 = L a$$

Compressive and tensile forces are equal and opposite

$$F_{c1} = F_{c2} = F_{T1} = F_{T2} = L$$

$$M_1 = M_2 = M_3 = M_4 = L a$$

$$+\uparrow F_y = 0 = j_y - i_y - F_{T2} + \frac{L a}{F} - (\frac{L a}{F} - L) - F_{T2} = 0$$

$$F_{T2} = L$$