Investigating the Effect of Deformation and Stirring on Pure Gold Surface Tension Measurements Using a Faraday Forcing Technique

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Outline

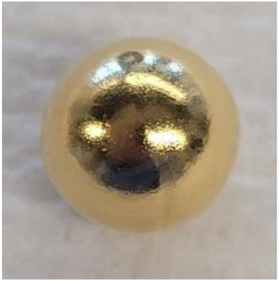
1. Motivation & Background
2. Experimental Methods
3. Effect of Deformation on Surface Tension
4. Effect of Stirring on Surface Tension
5. Uncertainty Analysis
6. Conclusions & Future Work

1. Motivation & Background

Why Study Surface Tension?

Motivation

- Study the validity of Faraday forcing in the ISS-EML
- Provides a benchmark dataset for the comparison of experimental techniques
- Surface tension governs melt-pool dynamics in additive manufacturing



Final Gold Sample Used in Batch 4

Key Challenge

EML testing induces large sample **deformation** and **internal stirring** that bias surface tension measurements.

Containerless Processing: ISS-EML vs. ESL

ISS-EML (This Study)

- Aboard the International Space Station
- Sample size
  6–10 mm / 2–4 g
- Pulse Excitation is used heavily

MSFC ESL (Comparison)

- Ground-based electrostatic levitation
- Sample Size
  1.5–3 mm / 30–50 mg
- Faraday excitation is the *standard* method

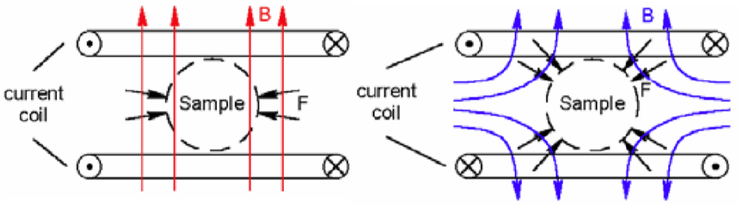
Novel Contribution

This study is the **first large-scale application of Faraday excitation in an ISS-EML environment**.

ISS-EML Coil Comparison

Comparison of Heater and Positioner Coils in ISS-EML

Feature	Heater Coil	Positioner Coil
Field type	Dipole	Quadrupole
Primary purpose	Heat & stimulate	Levitate & position
Force on sample	Equatorial squeeze	Symmetric
Deformation effect	High — drives oscillation	Low
Stirring effect	High — CCW vortex	Clockwise vortex



Heater field (red) and Positioning field (blue)

Surface Tension Theory

Eötvös Rule — surface tension decreases linearly with temperature

Rayleigh Equation — links oscillation frequency to surface tension:

$$\boxed{\gamma = \frac{3}{8}\pi m \nu^2}$$

where m is mass and ν is the natural oscillation frequency.

3D View Top View Side View

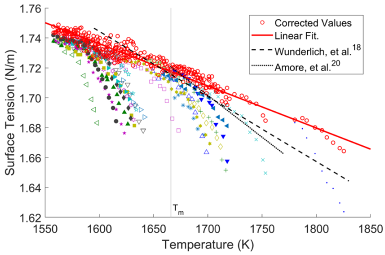
Ideal Droplet Mode Oscillation shown in 3D, the Top View and Side View

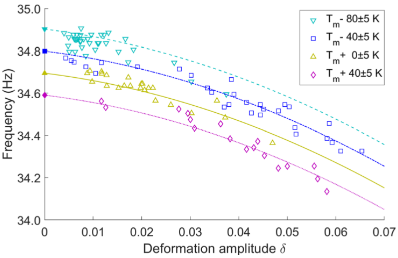
Deformation Induced Frequency Correction

Deformation Correction — at deformations $\gtrsim 1\%$ frequency decreases independently of temperature:

$$\boxed{\gamma = \frac{3}{8}\pi m \nu^2 (1 + p_1\delta + p_2\delta^2)}$$

where $\theta = \frac{T}{T_m}$ is dimensionless temperature, δ is deformation and p_θ, p_1 and p_2 are dependence factors





2. Experimental Methods

Faraday Excitation — A New Approach for EML

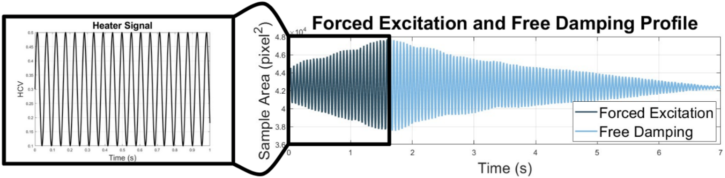
Faraday forcing: sinusoidal voltage applied to heater coil.

Optimised parameters:

- Frequency: 18.6 Hz
- Duration: 2.1 s
- Amplitude: 0.2 V

Why Faraday?

- Reduces the potential of other oscillation modes



Oscillation signal Including Excitation (Light Blue) Free Damping (Dark Blue) and Heater Voltage (Black)

Experimental Test Matrix

Experimental Test Matrix

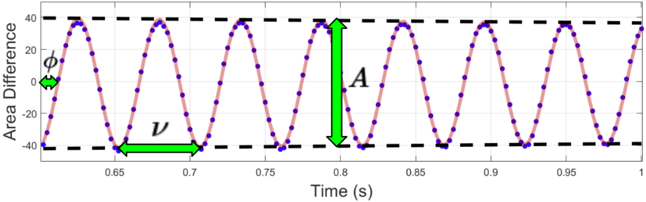
$U_{\mathrm{ctr}}^{\mathrm{H}}$ (V)	No. of Runs	Cycle Numbers
0.3	4	31, 32, 33, 34
0.5	1	36
0.6	2	37, 38†
0.8	1	39
1.0	1	40

- More runs at 0.3 V to characterise baseline and validate excitation parameters
- Positioner voltage held constant at 4.0 V across all runs

†Run 37 discarded (noisy signal); run 35 excluded (solidification during oscillation).

1. **Data segmentation** — overlapping 0.4 s windows (80% overlap) fit with Levenberg-Marquardt nonlinear regression:

$$y = Ae^{-t/\tau}\sin(2\pi\nu t + \phi) + D$$



Signal, y, with Visualization of Fitted Parameters

2. **Frequency correction** — fit corrected frequency formula to all (ν, δ, θ) data to extract p_1, p_2.

3. **Surface tension** — apply $\gamma = \frac{3}{8}\pi m\nu^2(1 + p_1\delta + p_2\delta^2)$ to all segments

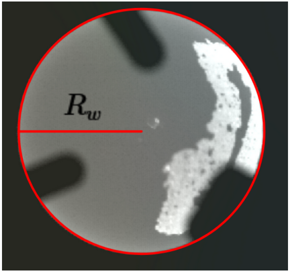
Deformation Definition

Top View

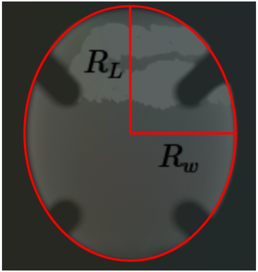
- $R_w = R_0(1 \pm \delta)$.
- $A = \pi R_w^2 - D$

Side View

- $R_L = \frac{R_0}{(1 \pm \delta)^2}$, $R_w = R_0(1 \pm \delta)$
- $A = R_L - R_w$



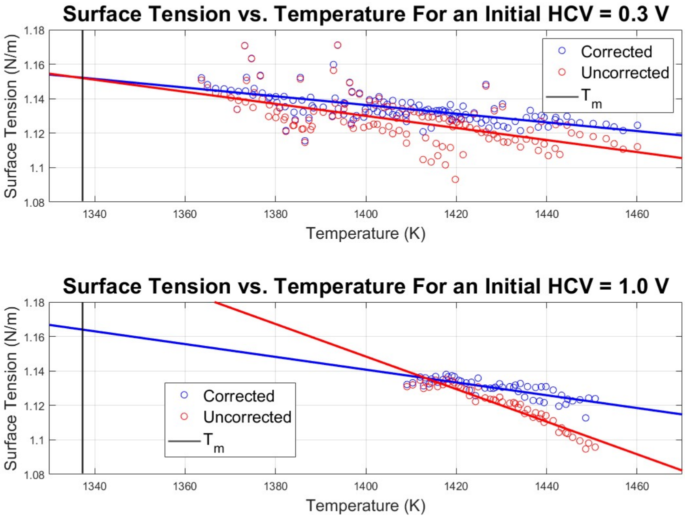
Top View Deformation



Side View Deformation

3. Effect of Deformation on Surface Tension

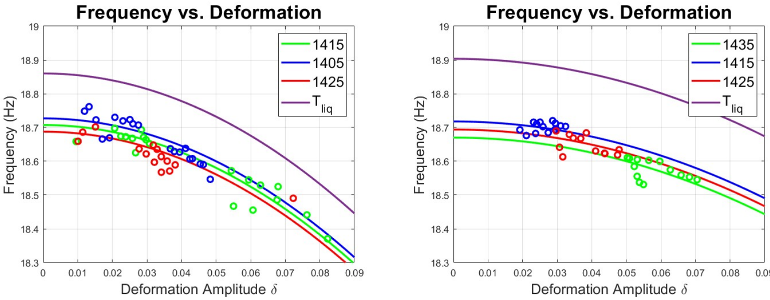
Corrected Surface Tension Results



Uncorrected Surface Tension Curves (Red) and Corrected Surface Tension Curves (Blue) for 0.3 V and 1.0 V

Deformation Increases with Heater Voltage

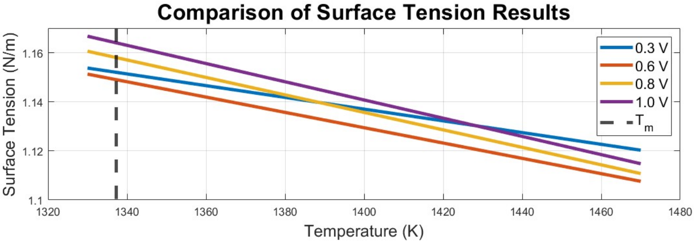
- Steady state deformation measured immediately before excitation increases monotonically with $U_{\mathrm{ctr}}^{\mathrm{H}}$
- Frequency maps **quadratically** to deformation — consistent with Xiao et al. (LEK-94 alloy).



Frequency (Hz) vs. Deformation For U_{ctr}^{H} of 0.3 V and 1.0 V

Deformation Corrected Surface Tension Results

Linear fits $\gamma = m(T - T_m) + b$ after deformation correction:



Comparison of Surface Tension Lines of Best Fit with U^H_{ctr} of 0.3 V (Blue), 0.6 V (Orange), 0.8 V (Yellow) and 1.0 V (Purple)

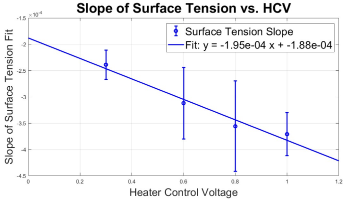
- Higher $U_{\mathrm{ctr}}^{\mathrm{H}}$ $\Rightarrow$ stronger temperature dependence (larger $|m|$)
- Higher $U_{\mathrm{ctr}}^{\mathrm{H}}$ $\Rightarrow$ higher surface tension at T_m (larger b)
- Correction significantly reduces temperature dependence vs. uncorrected results

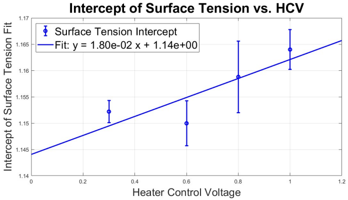
4. Effect of Stirring on Surface Tension

Statistical Evidence: Stirring Affects Surface Tension

χ^2 test run on corrected surface tension fit parameters:

Parameter	p-value	Conclusion
Slope m	0.0479	Statistically significant
Intercept b	0.0411	Statistically significant



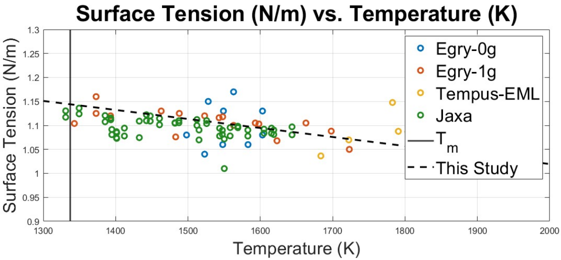


Extrapolation of Linear Fit Parameters (m, b) to Zero Stirring

Extrapolation to Zero Stirring

Linear extrapolation of m and b vs. $U_{\mathrm{ctr}}^{\mathrm{H}}$ to $U_{\mathrm{ctr}}^{\mathrm{H}} = 0$:

Parameter	Extrapolated Value
Slope	-0.000188 ± 0.000035 N/m·K
Intercept	1.145 ± 0.0062 N/m



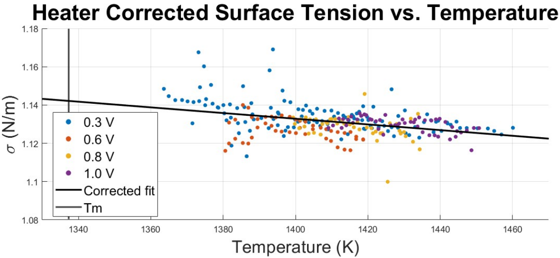
Comparison of Extrapolated zero stirring Results to Previous EML Studies with Pulse Excitation

Additive Stirring Correction Formula

A correction can be applied to remove stirring effect bias.

$$\gamma_C = \gamma_{\text{measured}} \; - \; m_{\text{slope}} \, U_{\text{ctr}}^{\text{H}}(T - T_m) \; - \; b_{\text{slope}} \, U_{\text{ctr}}^{\text{H}}$$

$$m_{\text{slope}} = -2.479 \times 10^{-4}, \quad b_{\text{slope}} = 1.712 \times 10^{-2}$$



Stirring Corrected Surface Tension for All Runs
$m = 1.484310^{-4} \pm 2.0910^{-5}, \quad b = 1.1421 \pm 1.6110^{-3}$

Lower coefficient of variance, $CV = \frac{u_b}{b}$ lower for Faraday forcing (CV $= 0.14\%$) than pulse excitation (CV $= 0.19\%$)

5. Uncertainty Analysis

Key Uncertainty Sources

Source	Relative Uncertainty (%)
Temperature, u_T/T	0.053
Mass, u_m/m	0.0018
Frame rate (top view, 150 Hz), $u_{\text{fps}}/\text{fps}$	6.23
Frame rate (side view, 400 Hz), $u_{\text{fps}}/\text{fps}$	2.33
Frequency (top), u_ν/ν	6.34
Frequency (side), u_ν/ν	2.63
Surface tension (top), u_γ/γ	12.68
Surface tension (side), u_γ/γ	5.26

Dominant Uncertainty

Camera **frame rate** dominates frequency uncertainty, which in turn dominates surface tension uncertainty via the quadratic dependence $\gamma \propto \nu^2$.

6. Conclusions & Future Work

Conclusions

Faraday excitation is valid in EML environments.

Free-damping frequencies are independent of the excitation frequency; results align with prior gold surface tension measurements. Lower coefficient of variance than Pulse excitation.

Stirring Substantially Affects Surface Tension Derivation

Evidenced by χ^2 statistical test and allows for an additive correction formula to be produced for prior surface tension results.

Big Picture

The inherent biases in the Faraday Forcing Technique have been investigated and corrected for. This shows the feasibility of the method and provides a benchmark dataset for other experimental initiatives.

Future Work

Deformation Modelling

Compare the Xiao et al. deformation correction with the "volume moved" metric of Fahimi et al. — may improve correction accuracy.

Improving the Heater Correction

Current correction is based on 4 data points. Extend the $U_{\mathrm{ctr}}^{\mathrm{H}}$ range (especially above $1.0\,\mathrm{V}$) to verify the linearity assumption.

Viscosity Measurement via Faraday Excitation

Viscosity has a stronger stirring dependence than surface tension. Faraday excitation features a less variable Reynolds number than pulse excitation, which makes it a candidate for precise viscosity measurements.

Acknowledgments

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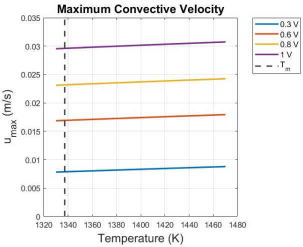
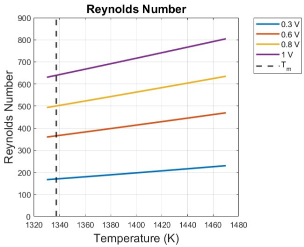
Quantifying Internal Stirring via Reynolds Number





Plots of Maximum Convective Velocity and Reynolds Number vs. Temperature for all Heater Control Runs

Flow Regimes by Re

Flow Regime	Re
Laminar	< 500
Transitional	$500 - 600$
Turbulent	> 600

Flow Regimes by U_{ctr}^{H}

$U_{\text{ctr}}^{\text{H}}$ (V)	Flow Regime
0.3, 0.6	Laminar
0.8	Transitional
1.0	Turbulent

Determining Deformation Correction Factors

The corrected frequency formula contains four unknowns fit via Levenberg-Marquardt nonlinear regression across all (ν, δ, θ) data for a given $U^{\mathrm{H}}_{\mathrm{ctr}}$:

$$\nu_c = \nu_{\theta=0}(1 + p_\theta \theta)(1 + p_1 \delta + p_2 \delta^2)$$

Fitted parameters:

- $\nu_{\theta=0}$ — natural frequency at liquidus temperature
- p_θ — temperature weight (constrained < 0; inverse ν–T relation)
- p_1 — linear deformation weight (constrained ≤ 0; positive values unphysical)
- p_2 — quadratic deformation weight (bounded per Xiao et al.)

Deformation Formula — Top View

The top camera observes changes in the **equatorial radius** R_w. For deformation δ:

$$R_w = (1 \pm \delta) R_0$$

The curve-fit parameter D equals the mean cross-sectional area (πR_0^2) and $|A| + D$ equals the peak deformed area:

$$\sqrt{\frac{|A| + D}{\pi}} = (1 \pm \delta)\sqrt{\frac{D}{\pi}}$$

Solving for δ:

$$\boxed{\delta = \sqrt{\frac{|A| + D}{D}} - 1}$$

where A and D come from the damped sine fit $y = Ae^{-t/\tau}\sin(2\pi\nu t + \phi) + D$.

Uncertainty: A and D are correlated — their covariance term $C_{1,5}$ from the fit covariance matrix must be included in error propagation.

Deformation Formula — Side View

The side camera observes both polar (R_L) and equatorial (R_w) radii. Define:

$$R_L = (1 \pm \delta)^{-2} R_0, \qquad R_w = (1 \pm \delta) R_0$$

The fit parameter $A = R_L - R_w$ gives:

$$A = \frac{R_0}{(1+\delta)^2} - R_0(1+\delta)$$

Applying the Taylor expansion $\frac{1}{(1+\delta)^2} \approx 1 - 2\delta + 3\delta^2$:

$$\frac{A}{R_0} = 3\delta^2 - 3\delta$$

Taking the physically meaningful (smaller) root of the quadratic:

$$\boxed{\delta = \frac{1 - \sqrt{1 - 4A/(3R_0)}}{2}}$$

Fluid Velocity — Surrogate Model

Internal droplet velocity is computed by subtracting positioner from heater contributions:

$$u_{\max} = \underbrace{\sum_{i,j,k,s} q_{ijks} \left(U_{\mathrm{ctr}}^{\mathrm{H}}\right)^i \rho^j (\ln \mu)^k (\ln \sigma_{\mathrm{el}})^s}_{\text{heater velocity}} - \underbrace{\sum_{i,j,k,s} p_{ijks} \left(U_{\mathrm{ctr}}^{\mathrm{P}}\right)^i \left(\frac{1}{\sqrt{\sigma_{\mathrm{el}}}}\right)^j \rho^k (\ln \mu)^s}_{\text{positioner velocity}}$$

Material properties (evaluated at each temperature step, 1330–1440 K):

Property	Expression	Source
Viscosity μ	$\mu_0 \exp\left(\frac{2.65\, T_m}{RT}\right)$, $\mu_0 = 0.5798\,\mathrm{mPa{\cdot}s}$	Literature
Density ρ	$17233 - 1.168(T - T_m)\,\mathrm{kg/m^3}$	Measured
Conductivity σ_{el}	$(2.500 \times 10^{-7} + 4.937 \times 10^{-11}(T - T_m))^{-1}\,\mathrm{S/m}$	Measured

Reynolds number then follows:

$$\mathrm{Re} = \frac{\rho\, u_{\max}\, d}{\mu}, \qquad d = \left(\frac{6m}{\pi \rho}\right)^{1/3}$$

Uncertainty Propagation Formulas

General propagation:

$$u_{\text{total}} = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)}$$

Temperature (coupled T_P, T_{PL} terms; $u(T_L) = 0$ for gold):

$$\frac{u_T}{T} = \sqrt{\left(\frac{1}{T_P} \left(\frac{u(T_P)}{T_P} \right) + \frac{1}{T_{PL}} \left(\frac{u(T_{PL})}{T_{PL}} \right) \right)^2 + \frac{1}{T_L^2} \left(\frac{u(T_L)}{T_L} \right)^2}$$

Frequency (frame-rate dominates):

$$\frac{u_\nu}{\nu} = \sqrt{\left(\frac{u_r}{r} \right)^2 + \left(\frac{u_{\text{fit}}}{\nu} \right)^2 + \left(\frac{\nu}{2 \cdot \text{fps}} \right)^2}$$

Uncertainty Propagation Formulas

Surface tension (uncorrected):

$$\frac{u_\gamma}{\gamma} = \sqrt{\left(\frac{u_m}{m}\right)^2 + \left(2\frac{u_\nu}{\nu}\right)^2}$$

Surface tension (corrected, excluding p_1/p_2 uncertainty):

$$\frac{u_{\gamma,c}}{\gamma_c} = \sqrt{\left(\frac{u_m}{m}\right)^2 + \left(2\frac{u_\nu}{\nu}\right)^2 + \left(\frac{1}{\gamma_c}\frac{\partial\gamma_c}{\partial\delta}\right)^2\left(\frac{u_\delta}{\delta}\right)^2} \quad \text{where} \quad \frac{1}{\gamma_c}\frac{\partial\gamma_c}{\partial\delta} = \frac{p_1 + 2p_2\delta}{1 + p_1\delta + p_2\delta^2}$$

Linear regression (slope m, intercept b):

$$u_m = \frac{s_{yx}}{\sqrt{\sum(x_i - \bar{x})^2}}, \qquad u_b = s_{yx}\sqrt{\frac{1}{N} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2}}, \qquad s_{yx} = \sqrt{\frac{\sum y_i^2 - b\sum y_i - m\sum x_i y_i}{N - 2}}$$

Surface Tension Results: Uncorrected vs. Corrected

Uncorrected

$U^{\mathrm{H}}_{\mathrm{ctr}}$	m (N/m·K)	b (N/m)
0.3	-0.000397 ± 0.000054	1.1563 ± 0.0043
0.6	-0.000697 ± 0.000076	1.1679 ± 0.0048
0.8	-0.000776 ± 0.000092	1.1856 ± 0.0074
1.0	-0.000945 ± 0.000043	1.2076 ± 0.0040

Corrected

$U^{\mathrm{H}}_{\mathrm{ctr}}$	m (N/m·K)	b (N/m)
0.3	-0.000201 ± 0.000038	1.1528 ± 0.0030
0.6	-0.000312 ± 0.000068	1.1500 ± 0.0043
0.8	-0.000356 ± 0.000086	1.1588 ± 0.0068
1.0	-0.000371 ± 0.000041	1.1640 ± 0.0038

Key Takeaway

Deformation correction reduces the temperature dependence $|m|$ at all voltages. The intercept b is also affected

Chi-Squared Test — Details

Hypothesis: fit parameters (m or b) are independent of heater control voltage (i.e., stirring has no effect).

Test statistic with $k = 4$ voltage levels and $\nu = k - 1 = 3$ degrees of freedom:

$$\chi^2 = \sum_{i=1}^{k} \frac{(\theta_i - \bar{\theta})^2}{u_{\theta_i}^2}$$

where θ_i is the fitted parameter at each $U_{\mathrm{ctr}}^{\mathrm{H}}$ and u_{θ_i} is its uncertainty from linear regression.

Significance level: $\alpha = 0.05 \quad \Rightarrow \quad$ critical value $\chi^2_{0.10,\,3} = 7.8$

Parameter	χ^2 statistic	χ^2_{crit}	p-value	Conclusion
Slope m	$> \chi^2_{\text{crit}}$	6.251	0.0179	Reject H_0 — stirring affects m
Intercept b	$> \chi^2_{\text{crit}}$	6.251	0.0514	Reject H_0 — stirring affects b

Rayleigh Equation — Full Derivation

Starting from the angular frequency of a levitated droplet:

$$\omega^2 = \ell(\ell - 1)(\ell + 2)\frac{\gamma}{R^3\rho}$$

For $\ell = 2$, $\omega = 2\pi\nu$, and substituting $m = \frac{4}{3}\pi R^3\rho$:

$$(2\pi\nu)^2 = \frac{8\gamma}{R^3\rho} \implies \boxed{\gamma = \frac{3}{8}\pi m\nu^2}$$

The $\ell = 2$ mode oscillates between prolate and oblate ellipsoidal shapes.